NEW YORK UNIVERSITY
INSTITUTE OF MATHEMATICAL SCIENCES
LIBRARY
25 Waverly Place, New York 3, N. Y.

# AEC Computing and Applied Mathematics Center

## AEC RESEARCH AND DEVELOPMENT REPORT

NYO-8672

SHERWOOD PROGRESS REPORT

No. 2

(July 1957 to July 1958)

by

H. Grad and J. Berkowitz

July 31, 1958

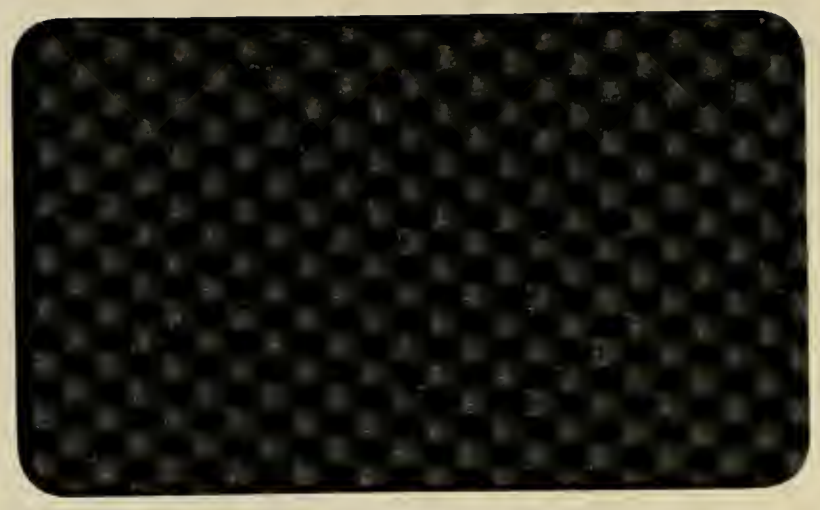
NEW YORK UNIVERSITY
INSTITUTE OF MATHEMATICAL SCIENCES
LIBRARY
25 Waverly Place, New York 3, N. Y.

## Institute of Mathematical Sciences

NEW YORK UNIVERSITY

NEW YORK, NEW YORK

NYO-8672



This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission to the extent that such employee or contractor prepares, handles or distributes, or provides access to, any information pursuant to his employment or contract with the Commission.

AEC COMPUTING AND APPLIED MATHEMATICS CENTER
Institute of Mathematical Sciences
New York University

NYO-8672

# SHERWOOD PROGRESS REPORT

No. 2

(July 1957 to July 1958)

by

H. Grad and J. Berkowitz

July 31, 1958

NEW YORK UNIVERSITY
INSTITUTE OF MATHEMATICAL SCIENCES
LIBRARY

Contract No. AT(30-1)-1480

## Preface

This report is a summary of activities carried on at the Institute of Mathematical Sciences at New York University from July 1957 to July 1958. It contains some personnel data, a summary of scientific progress during the past year, a list of reports and papers issued, and finally a list of references.

Personnel

The average number of equivalent full-time scientific staff members is 16.6 theoreticians and 4 experimenters. The number of individuals involved is somewhat greater. Of these, 8 have faculty rank or equivalent senior positions. There are 12 Ph.D.'s in mathematics, and 6 with Ph.D.'s in physics or other fields. Also employed are 8 research assistants. During the year 3 consultants have been used.

---

Shock Theory [18,26be,27ab]

The evidence has become increasingly strong that there do exist steady shock profiles with an irreversible transition from one constant state to another without the intervention of collisions as a damping mechanism. Most of the investigation has concerned a range of parameters in which a low temperature, low $\beta$* plasma is heated to a much higher temperature at a moderate $\beta$ by a shock of moderate strength.** It was established previously that such a shock, provided that it is observable within reasonable apparatus dimensions, heats the ions almost exclusively. The wave length of the oscillating shock profile is on the order of $\lambda=\sqrt{m/\mu ne^2}$ *** and the entire thickness of the transition can range from several times $\lambda$ to many times $\lambda$, depending on the parameters. A considerable amount of analytical work has been done involving all the general features. Exact quantitative results require numerical computation of an extremely intricate and non-standard type. This program is underway.

---

*** A representative value of $\lambda$ is .02 cm. at $n=10^{15}/cm^3$.

** Strength is measured by magnetic field ratio.

* $\beta=2\mu p/B^2$ = gas energy/magnetic energy.

Runaway Ions and Thermonuclear Reaction Rates [12,19,26d]

It was remarked at the Princeton meeting (Oct.'57) that, because of a dip in the ion dynamical friction curve occurring at fast ion speeds intermediate between the mean ion and mean electron speed, one might expect semi-runaway ions to develop on application of a moderate electric field. Subsequent quantitative analysis showed[12,19,26d] that no such effect exists, because the ion distribution remains very accurately Maxwellian even with an applied E, and the dip in the friction curve remains far out in the tail of the ion distribution. However, computation of the (small) perturbation of the ion distribution from Maxwellian leads to a corrected thermonuclear reaction rate. The increased rate can be quite large under certain circumstances, and a center of mass motion opposite to the applied E is predicted. It is not yet clear whether these effects are observable. From this analysis, it appears that actual runaway ions (with speeds greater than the mean electron speed) cannot appear under any conditions which are experimentally realizable. In computing the perturbation of the ion distribution, mathematical methods of computing the tail end of the distrubution were developed which may have many other applications (e.g. to computation of runaway electrons).

Theory of the Boltzmann Equation

A very complete set of transport coefficients has been computed for a fully ionized gas.[13,27h,31] These include many more coefficients than those of the Chapman-Enskog theory, e.g. a 2 x 2 matrix of heat conductivities (connecting electron and ion temperatures with electron and ion carried heat flows), a similar multi-

plicity of viscosity coefficients, thermal diffusion coefficients, together with all the additional "Hall" effects and the distinction between parallel and perpendicular values of each coefficient in the presence of a magnetic field. These coefficients include all the standard values of electrical conductivity. Much work has to be done to assess these results and discover under what circumstances the Chapman-Enskog values are inapplicable.

From the moment analysis of the Boltzmann equation one can derive a set of macroscopic fluid-like equations which contains the effect of finite Larmor radius together with anisotropic stress tensor and heat flow and current flow along the field lines. These equations should be considerably more accurate than the guiding center (or Chew-Goldberger-Low) equations. They are probably easier to apply than the zero Larmor radius self-consistent field technique and, in addition, take into account some of the effects of finite Larmor radius. So far, they have been applied to only the shock problem* with, as yet, inconclusive results[32].

The validity of the Boltzmann and Fokker-Planck equations has been studied[30]. It appears that, despite the fact that the two-particle correlation distance (viz. the Debye length) is much larger than the mean interparticle separation distance, the one-particle Boltzmann equation can be used (raising one's-self by one's boot straps) to compute the correct non-equilibrium shielding potential. Preliminary results have been computed. A tentative

*Preliminary results were given by M. H. Rose in Gatlinberg, June 1956.

correction of the conventional Boltzmann equation (with Coulomb potential cut off at the Debye distance) using the spherically symmetric shielding potential, $e^{-r}/r$, has been computed[15,26a].

Spiraling of Particles [11]

The constancy of the magnetic moment $\mu$ was demonstrated formally by Alfven to first order, by Hellwig to second order, and by Kruskal to all orders. These formal expansions have been shown to be actually convergent to solutions of the equations of motion. The problem is to estimate the error made in terminating the expansion at a given point and is a difficult one because of the possibility of cummulation of error after many Larmor cycles.

Self Consistent Field Problems

Techniques have been developed for solving by approximate means problems of much more generality than are accessible by exact methods. A specific problem that has been treated this way is the rotating mirror or homopolar machine. Once the self-consistent equilibrium solution has been found, the particle losses can be computed. Some results were presented at the theoretical meeting at N.Y.U. in May[27g]. The techniques involve a combination of the previously developed generalized adiabatic invariance theory[33], the guiding center fluid theory[34], together with certain other techniques for treating rotating systems[29].

Stability

A large number of mirror machine geometries with end plates were analyzed for stability using a simple sufficient condition[17, 26c,27d]. Stable configurations were found with interesting values

of $\beta$. The most interesting ranges of the parameters (viz. mirror ratio and aspect ratio) will be recomputed using the more complicated necessary and sufficient analysis.

The fact that a mirror machine has open ends, past which waves can pass remaining energy from the system, is a stabilizing influence that has not been previously analyzed. A rough analysis of this problem yields a finite amplitude of oscillation in systems which would otherwise be called unstable.

Cusped Geometries

Methods of computing three dimensional cusped geometries both analytically and numerically are being developed. Particle losses from low $\beta$ picket fence geometries are being computed in order to have precise results for comparison with cusped geometries. In many respects, the theory has been carried as far as that which is available for other devices, and no large theoretical program is warranted without experimental guidance.

Experimental Program

The laboratory was transferred to new quarters with greater space.

An automatic charging and firing system was designed and built with improved safety features, permitting experiments to be performed in five seconds instead of the previous 100 seconds or more. This has materially increased capacitor lifetime. In addition , the main capacitor bank was redesigned for horizontal operation and has been fired repeatedly at three kilovolts (maximum voltage) without breakdown.

A new vacuum system was put into operation, permitting experiment pressures down to $10^{-6}$ mm Hg. A new mass spectrometer leak detector system has been installed also.

New rail and coaxial sources have been constructed, employed up to ten kilovolts; the coaxial source has evidenced more uniform plasma burst alignment.

With this equipment, successful plasma constrictions (pipes) up to one meter have been obtained in both two inch and one inch straight glass tubes. Conversely, U-bend studies show that in regions of high field curvature the plasma flow curvature remains small. However, a tube of larger radius of curvature has been constructed which will employ greatly increased fields.

Time-of-flight measurements were performed on the plasma in the one inch tube. Results indicated plasma velocities of over two centimeters per microsecond, essentially supersonic.

A large series of experiments were performed in the one inch tube, shooting plasma bursts along axial fields, up to twenty kilogauss, which has been distorted or diverted by brass obstacles. The plasma appeared to follow the diverted field lines as predicted by theory.

A report[16] was prepared and distributed of all of the important results obtained since July 1, 1957.

A new sixty kilovolt coaxial plasma source (cannon) was recently completed, together with its own vacuum spark gap trigger assembly. For the present, it is being employed at twelve kilovolts (twenty times the previous source energy) to gather preliminary

data, and to avoid the possibility of destructive testing.

A new two inch straight experiment tube was constructed on which the center third of the main field solenoid was wound at four times the pitch of the outer thirds, thus (theoretically) increasing the flux density in this region four-fold. This tube was assembled with the new cannon source. Preliminary results show conclusively that the plasma converges in the central region where the field lines would be expected to converge.

A detailed time-of-flight study has been almost completed on the above system. Preliminary results suggest that the plasma burst expands longitudinally since the leading and trailing edges evidence different velocities. If this velocity difference were interpreted as temperature it would indicate thermal energies in excess of ten, and possibly one hundred, volts.

MH-V (Ref. [5])

This report contains material which was to a large extent carried out in 1954 and summarized at the Princeton meeting, 1954. The mathematical theory of Maxwell's equations without displacement current is developed extensively and is found to be considerably simpler and more easily applied than that of the full Maxwell's equations, e.g. in the transient theory of the pinch. This theory (which is Galilean invariant) is, in a sense, more appropriate than the theory of the full equations, when used together with non-relativistic fluid equations.

MH-VI (Ref. [6])

This report contains background material in magnetohydrodynamics derived and collected for easy reference in later reports.

MH-VII (Ref. [7])

In this paper a number of exact mathematical analogues of magnetohydrodynamic problems with conventional fluid dynamical problems are presented. This makes available a wealth of literature and experience for use in these special problems. This material was developed, in large part, in 1954, and was issued in a mimeographed set of NYU lecture notes in 1954.

MH-VIII (Ref. [8])

This contains an elaboration and extension of the material contained in LAMS-2105, issued in 1957. The theory of the complete set of nonlinear classical magnetohydrodynamic equations

(including Maxwell's equations without displacement current) is developed and many techniques of gas dynamics are found to be directly applicable. Small disturbances and hydromagnetic shocks are treated, as well as the theory of "simple waves".

Ref. [11]

Discussed above.

Ref. [12]

Discussed above. There is also included an analysis of the relation between the Boltzmann and Fokker-Planck equations.

Ref. [13]

Discussed above.

Ref. [14]

The magnetohydrodynamic theory of the full Maxwell's equations coupled with a set of relativistic fluid equations is shown to be mathematically equivalent to the theory of the conventional equations (MH-VIII).

Ref. [15]

Discussed above.

Ref. [16]

Discussed above.

Ref. [17]

Contains a complete account of the theory of stability of free boundary equilibria treated by variational methods [35,27d,246e], also the analysis of a special problem with rotating magnetic fields; also the mirror machine stability computation referred to above.

Ref. [18]

Discussed above.

Ref. [19]

Discussed above.

Ref. [20]

Contains a summary of the theory of magnetohydrodynamic equilibrium (of which force free fields are a special case). The results presented embrace considerably more material than exists in the previously published literature.

Ref. [21]

Contains a condensation of the theory of cusped geometries.

## References


Notes on Magneto-Hydrodynamics, NYO-6486.

[1] MH-I, General Fluid Equations, H. Grad, August 1, 1956.

[2] MH-II, Dimensional Considerations, (Part 1), H. Grad, and (Part 2) M. H. Rose, Aug. 7, 1956.

[3] MH-III, Special Solutions, H. Grad, Aug. 14, 1956.

[4] MH-IV, Ohm's Law, H. Grad, Aug. 20, 1956.

[5] MH-V, Theory of Maxwell's Equations without Displacement Current, A. A. Blank, K. O. Friedrichs, and H. Grad, Nov. 1, 1957.

[6] MH-VI, Fluid Magnetic Equations - General Properties, H. Grad and A. A. Blank, July 1, 1958.

[7] MH-VII, Fluid Dynamical Analogies, A. A. Blank and H. Grad, July 15, 1958.

[8] MH-VIII, Nonlinear Wave Motion, K. O. Friedrichs and H. Kranzer, July 31, 1958.

[9] On the diffusion of a Conducting Fluid across A Magnetic Field, M. H. Rose, July 1, 1956, NYO-7691.

[10] On the Structure of a Steady Hydromagnetic Shock: One Fluid Theory, M. H. Rose, Oct. 1956, NYO-7693.

[11] On the Asymptotic Series Expansion of the Motion of a Charged Particle in Slowly-varying Fields, J. Berkowitz and C. S. Gardner, Dec. 23, 1957, NYO-7975.

[12] Thermonuclear Reaction Rates in an Electrical Discharge, H. Grad, Jan. 23, 1958, NYO-7977.

[13] Moment Description of Gas Mixtures I, I. I. Kolodner, Sept. 10, 1957, NYO-7980.

[14] Basic Notions of Relativistic Hydromagnetics, P. Reichel, Apr. 15, 1958, NYO-7697.

[15] Computation of the Transport Coefficients Using the Shielded Coulomb Potential, R. Liboff, June 20, 1958, NYO-8669.

[16] Experiments on Supersonic Plasma Flow along Magnetic Fields, D. Finkelstein, M. Ehrlich, I. Livingston and D. Wetstone, Mar. 27, 1958, NYO-7981.

Papers for Geneva, Sept. 1958

[17] Problems in Magnetohydrodynamic Stability, J. Berkowitz, H. Grad, and H. Rubin.

[18] Hydromagnetic Shock Waves in High-Temperature Plasmas, C. S. Gardner, H. Goertzel, H. Grad, C. S. Morawetz, M. H. Rose and H. Rubin.

[19] Thermonuclear Reaction Rates in a Discharge, H. Grad.

[20] Hydromagnetic Equilibria and Force Free Fields, H. Grad and H. Rubin.

[21] Cusped Geometries, J. Berkowitz, K. O. Friedrichs, H. Goertzel, H. Grad, J. Killeen and E. Rubin.

Cusped Geometries

[22] Theory of Cusped Geometries I - General Survey, H. Grad, Dec. 1, 1957, NYO-7969.

[23] Theory of Cusped Geometries II - Particle Losses, J. Berkowitz and H. Grad, to appear.

Papers Presented at Sherwood Meetings

[24] Berkeley, Feb. 1957, TID-7536.
- (a) Survey of the NYU Program, H. Grad, p.40.
- (b) Stability of Magnetic Mirror Machines, H. Rubin, p.193.
- (c) Morphology of Stable Geometries, H. Grad, p.196.
- (d) Particle Losses in Crossed Fields, J. Killeen, p,200.
- (e) Rigorous Justification of Kruskal's Asymptotic Series, C. Gardner and J. Berkowitz.
- (f) Shock Heating, H. Grad.
- (g) Shock Profiles, C. Morawetz.

[25] Harwell, June 1957.
- (a) Survey of NYU Activities, H. Grad.
- (b) Recent Developments at NYU, H. Grad.

[26] Washington, Feb. 1958.
- (a) Justification of Debeye Cut-Off, R. Liboff.
- (b) Steady State Shock Model, C. S. Morawetz.
- (c) Qualitative Considerations Concerning Stability, H. Grad.
- (d) Neutron Emission in a Discharge, H. Grad.

(e) Wave in Plasma in Magnetic Field Caused by Application of Crossed Electric Field, C. S. Gardner.
(f) Force Free Electromagnet inside a Spherical Shell, G. K. Morikawa.

[27] New York, May 1958
(a) Steady State Shock Model, C. S. Morawetz.
(b) Steady State Waves in a Plasma: Asymptotic Behavior at Infinity in One-Dimensional Rectilinear Geometry, J. Calkin.
(c) Stability of Relativistic Plasma Streams, D. Finkelstein.
(d) Stable Mirror Machines, H. Kranzer and H. Rubin.
(e) Explicit Equilibrium Configurations, H. Rubin.
(f) Reciprocal and Conjugate Vectors Applied to Equilibrium and Force-Free Fields, H. Rubin.
(g) Collisional Particle Losses in a Homopolar Device, H. Grad.
(h) Transport Coefficients of a Plasma by the Method of Moments, I. I. Kolodner.

[28] Sherwood Progress Report No. 1, Oct 1, 1957, NYO-7968.

[29] H. Grad, J.of Phys. Chem., 56, 1039 (1952).

[30] H. Grad, Nice conference, June 30, 1958 - July 6, 1958.

[31] I. I. Kolodner, NYU Thesis (1950).

[32] On the Structure of a Shock in a Magnetic Field, M. H. Rose, TID-7520 (Sept.1956), p.547.

[33] General Adiabatic Theory, H. Grad, TID-7520 (Sept.1956), p.402.

[34] A Guiding Center Fluid, H. Grad, TID-7503 (Feb.1956) p.495.

[35] H. Grad, WASH-184 (Jan.1955) p.144
WASH-289 (June 1955), Motion and Stability of Free Surfaces, p.115.

TID-7503, (Feb.1956), Stability of Free Surfaces, p.238.
TID-7520, (June 1956), Some New Developments in Stability of Free Surfaces, p.373.

[36] H. Rubin, Local Stability of an Incompressible Plasma Mixed with a Magnetic Field, TID-7503, (Feb.1956), p.247.
J. Berkowitz, Dynamical Stabilization of the Pinch, TID-7503, (Feb.1956), p.241.

MAR 2 1960

Date Due

AUG 10 '61

JUN 1 1963

PRINTED IN U. S. A.

NYU
NYO-
8672 Grad & Berkowitz.
Sherwood progress report
No. 2: (July 1957 to July 1958

NYU
NYO-
8672 Grad & Berkowitz.
AUTHOR
Sherwood progress report
TITLE No. 2: (July 1957 to
July 1958).

| DATE DUE | BORROWER'S NAME | ROOM NUMBER |
|---|---|---|
| AUG 10 '58 | [illegible] | |
| | [illegible] | |
| | [illegible] | |

N. Y. U. Institute of
Mathematical Sciences
25 Waverly Place
New York 3, N. Y.

NEW YORK UNIVERSITY
INSTITUTE OF MATHEMATICAL SCIENCES
LIBRARY
25 Waverly Place New York 3, N. Y.